radii of a sphere be drawn parallel to the radii of absolute curvature of any closed curve whatsoever, they will divide the sphere into two equal parts; for the proposed curve may be regarded as a geodetic line upon a surface so described that the tangent plane at any point along the given curve is perpendicular to the radius of absolute curvature at that point.

If the boundary curve be a loop of a geodetic line, the total curvature of the included portion of the surface is equal to a hemisphere diminished by the external angle of the loop.

If the boundary be a polygon whose sides are geodetic lines, the total curvature will be equal to a hemisphere diminished by the sum of the external angles of the figure. This proposition includes Gauss' celebrated theorem respecting the total curvature of a triangle formed on any surface with geodetic lines.

If the surface S be itself a sphere, we can represent the *area* of any closed curve B traced upon it by a plane angle. For this purpose, let a developable surface be circumscribed along the curve B, and let the angle be constructed as in the theorem. In this way we find the area of a small circle of the sphere to be equal to the defect by which the developed angle of the circumscribed cone falls short of four right angles.

The Rev. Professor Haughton communicated the following account of some barometric determinations of height made by him, with the view of examining by direct observation the different formulæ which have been proposed for introducing the hygrometric condition of the air into the calculation of heights:—

The uncorrected barometric formula is the following:—

$$H = 10000^{fath.}\left(1 + \frac{\Theta}{493}\right)\log\frac{p}{p'} \qquad \text{(I.)}$$

in which Θ denotes the mean excess of the temperature of the

column of air above 32°; p, p', the corrected barometric heights for the temperature 32°, at the lower and upper stations respectively; and H, the height of the mountain in fathoms.* If aqueous vapour constitute part of the column, and the mixture of dry air and vapour be in a state of equilibrium, then the column of air calculated by (I.) will be too short, and must be increased by the expansion due to the quantity of vapour in the column. If f, f' denote the elastic force of the vapour at the lower and upper stations respectively, it is easy to show that the hygrometric coefficient, by which equation (I.) should be multiplied, is

$$\frac{p+p'}{p+p'-f-f'};$$

This will convert the formula (I.) into the following:—

$$H = 10000^{fath.}\left(1+\frac{\Theta}{493}\right)\left(\frac{p+p'}{p+p'-f-f'}\right)\log\frac{p}{p'} \quad \text{(II.)}$$

This may be considered as the *statical* barometric formula, and in it account is taken of both moisture and temperature. It is certain that if the air be in a state of equilibrium, this formula will represent with accuracy the difference of level between the two stations.

If vapour or any other gas be suddenly introduced into a portion of a vertical column of air, and it requires *time* for the air to expand in consequence, the barometric pressure observed at any such point will be too great by a quantity, which at its maximum will be the elastic force of the vapour introduced. If we suppose therefore that the true barometric pressure at any point is $p-f$, this is equivalent to supposing that the introduced vapour has not yet commenced to expand the column. This supposition may be considered as belonging to the state of incipient motion. If we suppose that the expansion of the air, consequent on the introduction of the vapour, has ceased,

* The exact coefficient for the latitude of Dublin is 10008 fath. 2 ft.

and the air has returned to a state of equilibrium, then the observed barometric pressure is correct, and no deduction is to be made from it in consequence of the pressure of vapour. This is the supposition made in (II.), or the *statical* formula.

In the present state of our knowledge, I fear it is impossible to form a *dynamical* hygrometric correction for the barometric formula, but the principles on which it depends may be thus stated.

Let f denote the observed elastic force of the vapour at any point; this quantity is the sum of two elastic forces

$$f = f_s + f_d \tag{1}$$

f_s denoting that part of f which is doing *statical* work, i. e. bearing the *weight* of the vapour in the column; and f_d denoting that part of f which is doing *dynamical* work, i. e. lifting and expanding the column of air.

The barometric pressure at any point is therefore the sum of three quantities, viz., the pressure of the dry air, the statical pressure of the vapour, and the dynamical pressure of the vapour. Let ϖ denote the pressure of the dry air, then

$$p = \varpi + f_s + f_d \tag{2}$$

If the air be in equilibrium, $f_d = 0$, and $p = \varpi + f_s$, this is the value of p used in formula (II.): but if we suppose $f_s = 0$, i. e. the whole of the vapour at any point to be employed in moving the column, then $p = \varpi + f$, f_d becoming equal to f, the whole force of the vapour; but from (2) it is plain that $\varpi + f_s$ is the pressure to be used in the barometric formula; and in this supposed case $\varpi + f_s = \varpi = p - f$.

This corresponds to the case of incipient motion. Introducing it into (I.) we find

$$H = 10000^{fath.}\left(1 + \frac{\Theta}{493}\right) \log \frac{p-f}{p'-f'}; \tag{III.}$$

and from (II.) we obtain

$$H = 10000^{fath.}\left(1 + \frac{\Theta}{493}\right)\frac{p+p'}{p+p'-f-f'} \log \frac{p-f}{p'-f'}. \tag{IV.}$$

This latter formula (IV.) is the formula proposed by Dr. Apjohn (Proc. Royal Irish Academy, vol. ii. p. 561).

The dynamical formula may be thus written down, although unfortunately it cannot be used:—

$$H = 10000^{fath.}\left(1+\frac{\Theta}{493}\right)\frac{p+p'}{p+p'-f_s-f'_s}\log\frac{p-f_d}{p'-f'_d}. \quad \text{(V.)}$$

The statical part of the elastic force of the vapour appearing in the hygrometric coefficient, and the dynamical part under the logarithm.

If in (V.) we make $f_d=0$, $f'_d=0$, then since $f_s=f$, $f'_s=f'$, we obtain the statical formula (II.); if we suppose $f_s=0$ $f'_s=0$; then since $f_d=f$, $f'_d=f'$, we find (V.) reduced to (III.), the hygrometric coefficient disappearing, as it ought, for we have implicitly supposed that no expansion has yet taken place in the column of air. Equation (III.) may therefore be considered as the barometric formula corresponding to the state of incipient expansion. The heights calculated from (III.) will be in general smaller than those calculated from (I.) without hygrometric correction, because the elastic force of the vapour diminishes faster than that of the dry air, and therefore* the ratio of p to p' will be greater than of $p-f$ to $p'-f'$; consequently, the heights calculated from (III.) will be smaller than those deduced from (II.). The two corrections used in (IV.) tend to counteract each other, one increasing and the other diminishing the height, so that it sometimes happens that the heights calculated from (I.) and (IV.) are absolutely equal. It frequently occurs, however, from the irregular development of vapour at particular places, that the ratio of f to f' is less than of p to p', and, consequently, that the ratio of $p-f$ to $p'-f'$ is greater than of p to p'. In such cases, formula (III.) will give a greater height than (I.).

* If $\frac{f}{f'} > \frac{p}{p'}$, then $fp' > pf'$, or $fp' - pf' > 0$, and $pp' - pf' > pp' - fp'$, therefore $\frac{p}{p'} > \frac{p-f}{p'-f'}$; and *vice versâ*, if $\frac{f}{f'} < \frac{p}{p'}$.

In the following observations, the lower station was the N.W. coping-stone of the Barrow lock-gate in the town of Carlow; the upper station, the summit of Clogrenan Hill. Diff. of level by Ordnance Map = 157·00 fath.

TABLE I.

No. of Obs.	Lower Station. p	f	t	Upper Station. p'	f'	t'	Therm. Coeff. $1+\frac{\theta}{493}$	Hygrom. Coeff. $\frac{p+p'}{p+p'-f-f'}$	OBSERVATIONS.
	in.	in.							
1	29·654	0·260	50°	28·646	0·258	4 °	1·034	1·008	Oct. 2, 1849; fine day.
2	29·071	0·320	48	28·054	0·298	45	1·029	1·011	Oct. 3, 1849; wet, cloudy day.
3	29·362	0·234	50	28·355	0·222	44	1·033	1·008	Oct. 4, 1849; fine day.
4	29·555	0·271	47	28·518	0·263	42	1·025	1·009	October 5, 1849; heavy rain.
5	29·880	0·252	49	28·852	0·203	45	1·030	1·008	Oct. 8, 1849; fine day.
6	29·934	0·238	49	28·900	0·249	45	1·030	1·008	Oct. 9, 1849; fine day.
7	29·687	0·281	49	28·665	0·249	45	1·030	1·007	October 10, 1849; fine, cloudy.
8	29·651	0·214	51	28·620	0·217	46	1·033	1·007	October 12, 1849; high wind, fine.
9	29·591	0·430	59	28·581	0·420	54	1·050	1.015	October 19, 1849; wet and cloudy.
10	29·609	0·319	55	28·610	0·308	49	1·040	1·011	October 20, 1849; fine; high wind.

In observations 9 and 10, the lower station was at a point situated 15·06 feet above the lower station of the first eight observations. The barometer employed was made by Mr. Newman, of Regent-street. An observation of this barometer was made on setting out and returning from the Hill, and the exact height, at the time of the observation at the upper station, was found by interpolation, with the aid of observations of a good barometer, recorded by another observer, within a few yards of the lower station. In observations 9 and 10, the observation at the lower station was made simultaneously with a second barometer of Mr. Newman's construction.

In the following Table, I have calculated the heights from

the four formulæ; for the reasons already given, the fifth and most correct formulæ cannot be used in practice:—

TABLE II.

No.	I.	II.	III.	IV.	Range of Barom. from 10, A.M., to 4, P.M.
1	155·30	156·54	156·38	157·63	– ·029
2	159·13	160·88	157·39	159·12	+ ·013
3	156·56	157·81	155·93	157·18	+ ·094
4	159·00	160·43	159·22	160·65	+ ·026
5	156·61	157·86	150·30	151·50	+ ·044
6	157·25	157·51	160·25	161·53	– ·092
7	156·70	158·11	153·19	154·57	– ·061
8	158·76	159·87	160·41	161·53	+ ·034
9	160·87	163·28	159·12	161·50	– ·157
10	157·53	159·26	157·49	159 22	– ·069
	157·771	159·155	156·968	158·443	

On examining column I. of these observations, it is plain that they may be divided into two distinct groups, of which Nos. 1, 3, 5, 6, 7 are below the average, and Nos. 2, 4, 8, 9, 10 are above the average. Of the latter, Nos. 2, 4, 9 were made on wet days; Nos. 8, 10, on windy days, and in all, the state of the atmosphere may be considered as unsettled; although, so far as the change in the barometer is considered, Nos. 2, 4, 8 will bear comparison with the fine days.

If we take the mean results of the observations on settled and unsettled days, we obtain the following Table:—

TABLE III.

	I.	II.	III.	IV.
Settled, ...	156·484	157·566	155·210	156·482
Unsettled, ...	159·058	160·744	158·726	160·404

Column III. is less than I. for the reason already given. Comparing columns I. and IV., it is interesting to observe

how nearly they agree in settled weather, showing that the effect of the two hygrometric corrections is equal and opposite.

From the preceding observations, it appears that on wet days the barometric formula (II.), corrected statically for the hygrometer, gives too great a value for the height. As this fact does not appear to have attracted the attention of observers, it may be useful to confirm it by other cases which have been observed.

In the following observation of the height of Douce and Sugar Loaf, the lower station was at Kilmacanoge cross roads, at a point marked on the Ordnance Map as 255 feet, or 42·5 fathoms. Simultaneous observations were made with a Newman's barometer, which had been carefully compared with my own.

The lower station at Howth was the foot of the cliff in Balscaddan Bay:—

TABLE IV.

No.	Lower Station.			Upper Station.			Ther. Coeff.	Hygr. Coeff.	OBSERVATIONS.
	p	f	t	p'	f'	t'			
1. Douce, . .	29·663	0·372	58	27·421	0·361	50	1·044	1·014	Aug. 31, 1849; wet and foggy at summit of Douce.
2. Sugar Loaf,	29·635	0·372	58	28·153	0·278	51	1·045	1·011	Aug. 31, 1849; variable.
3. Howth, . .	29·684	0·440	63	29·103	0·419	58	1·058	1·915	June 28, 1852; wet day.

In order to compare the heights calculated from these observations with the trigonometric heights of the Ordnance Survey, we must add 42·5 fathoms for the height of the lower station in Nos. 1 and 2, and two fathoms for the height of the lower station in No. 3 above low water of spring tides. These corrections have been made in the following Table, in which V. denotes the trigonometric heights:—

TABLE V.

	I.	II.	III.	IV.	V.
Douce, ...	398·84	403·83	401·70	406·73	397·33
Sugar Loaf, .	275·32	277·88	263·03	265·45	275·17
Howth,. . . .	93·24	94·61	90·84	92·17	93·83

Column I., which is only corrected for temperature, is almost the same as V., and the figures in column II. are greater than V.

I shall add to these observations of my own three observations of the same height made by the Rev. Professor Jellett in the neighbourhood of Zermatt. I have calculated the two following Tables from the figures furnished by his note-book:—

The lower station is at Zermatt, the upper at the Schwarzsee:—

TABLE VI.

No.	Lower Station.			Upper Station.			Therm. Coeff.	Hyg. Coeff.	Range.
	p	f	t	p'	f'	t'			
1	24·856	0·379	59·5	22·128	0·294	53	1·049	1·014	+ ·053
2	24·880	0·355	59·5	22·207	0·260	50	1·046	1·013	– ·046
3	24·853	0·263	57	22·158	0·190	58	1·053	1·010	– ·024

The heights calculated by the four formulæ from these figures are—

TABLE VII.

	I.	II.	III.	IV.	
1	529·63	537·04	520·56	527·85	Fine.
2	516·30	523·01	504·52	511.08	Fine.
3	524·90	530·15	515·63	520.78	Fine.
Mean. .	523·61	530·06	513·57	519·903	

In this Table, the reduction of heights by formula III. is very striking; it is also remarkable that No. 3 of column II.,

on which day there was least moisture in the air, is the mean of the whole three observations.

Sir Robert Kane brought under the notice of the Academy the results of the analysis of the waters of the streams which descend from the side of the Dublin mountains, such as the Three Rock Mountain, with a view to illustrate the process of decomposition of the granite masses of those rocks, and the conversion of the felspathic elements into clays adapted for ceramic manufactures. A great number of springs and wells along the line of hills from Glencullen to Dundrum had been examined, and with similar results; but Sir Robert Kane specially detailed the quantitative analyses of two waters from Ticknock, above Rathfarnham, on the flank of the Three Rock Mountain.

The first of these specimens of water was taken from a rapidly running stream, and it was found that it contained a considerable quantity of soluble silica, combined with alkalies, there being both potash and soda present. This stream passed over a considerable tract of decomposing granite: 148,000 grains of this water left a residue on evaporation of 12·5 grains. This residue was found to contain the ordinary constituents of surface water, but in addition, alkalies and silicates amounting to—

Silica, ...	5,061	100,000.
Potash, ...	2,345	
Soda, ...	13,950	

The presence of alkaline silicates in such quantity in this water induced Sir Robert Kane to have a still more detailed analysis made of the water contained in a cavern excavated in one of the quarries made for obtaining what is called freestone, that is, the coarse powder of decomposed granite used in Dublin for scrubbing floors. This water was stagnant, and was derived from drainage through the adjoining masses of decompos-